ARTIFICIAL INTELLIGENCE, COMPUTATIONAL THINKING, AND MATHEMATICS EDUCATION

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Abstract

This paper examines the intersection of artificial intelligence (AI), computational thinking (CT), and mathematics education (ME) for young students (K-8). Specifically, I focus on three key elements that are common to AI, CT and ME: (a) agency, (b) modelling of phenomena, and (c) abstracting concepts beyond specific instances. Seeing ME through the lenses of other disciplines and recognizing that there is a significant overlap of key elements reinforces the importance of agency, modelling and abstraction in ME and provides new contexts and tools for incorporating them in classroom practice.

Introduction

In this paper I examine the intersection of artificial intelligence (AI), computational thinking (CT), and mathematics education (ME) for young students (K-8). Specifically, I focus on three key elements that are common to AI, CT and ME: (a) agency, (b) modelling of phenomena, and (c) abstracting concepts beyond specific instances (see Figure 1).

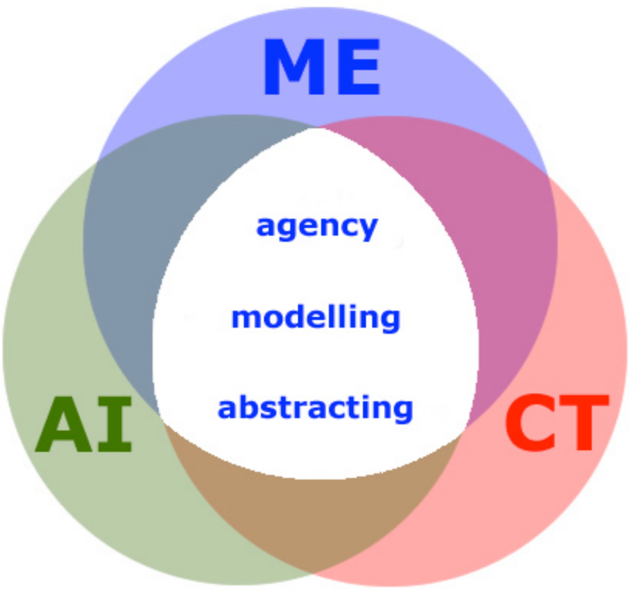

Figure 1. Three common elements of artificial intelligence, computational thinking, and mathematics education.

The theoretical framework of this paper adopts a sociocultural perspective where knowledge is constructed in interactions with others (Vygotsky, 1978). *Other*s also refers to the multiplicity of technologies that surround us, including both the digital artefacts of our new media world, and the human methods and specialized processes acting in the world. Technology is not simply a tool for human intention. It is an actor in the cognitive ecology of immersive humans-with-technology environments (Levy, 1993, 1998) that supports but also disrupts and reorganizes human thinking (Borba & Villareal, 2005). Actor-network theory (Latour, 2005) emphasizes the reciprocal relationship between the "actor" and technology, where we are both acting and acted upon (Thumlert, deCastell, & Jensen, 2014). In this examination of the overlap of AI, CT and ME, I identify and explore key elements of CT as actors we (can) think-with in the learning and teaching process.

The first two sections below briefly introduce AI and CT. The third section discusses how agency, modelling and abstraction may be seen as three common key elements of AI, CT and ME. The fourth section describes a proposed mathematics classroom project that integrates these elements and incorporates AI and CT.

Artificial Intelligence

AI is the intelligence evident in machines or software.

> It is also the name of the academic field of study which studies how to create computers and computer software that are capable of intelligent behavior. Major AI researchers and textbooks define this field as "the study and design of intelligent agents," in which an intelligent agent is a system that perceives its environment and takes actions that maximize its chances of success. ("Artificial Intelligence," n. d., para. 1)

Today, AI is increasingly pursued in a variety of ways by industry, such as seen in the development of self-driving cars by Google and cognitive systems like Watson by IBM.

AI Singularity

Some experts estimate that we are 20-50 years away from an AI singularity, where machines capable of recursive self-learning surpass human intellectual capacity and control.

AI machines that match and surpass human intelligence may be seen as leading to positive technological advances, such as eliminating aging and disease or enhanced space travel (Bostrom & Yudkowsky, 2014). At the same time, an AI singularity may prove disastrous. Stephen Hawking told the BBC (Cellan-Jones, 2014),"The development of full artificial intelligence could spell the end of the human race." Hawking (2014, para. 7) wrote:

> If a superior alien civilisation sent us a message saying, "We'll arrive in a few decades," would we just reply, "OK, call us when you get here – we'll leave the lights on"? Probably not – but this is more or less what is happening with AI. Although we are facing potentially the best or worst thing to happen to humanity in history, little serious research is devoted to these issues [...] All of us should ask ourselves what we can do now to improve the chances of reaping the benefits and avoiding the risks.

AI in Education

AI in education has historically focused on the design of digital tutors that not only provide exposition of concepts to be learned, but also have the intelligence to respond meaningfully to student behaviour, such as providing adaptive support (Gilbert, Blessing, & Guo, 2015), addressing student learning styles (Dorca, 2015), or providing culturally appropriate communication (Blanchard, 2015). Historically, these tutors were embedded in software packages designed for specific content areas, such as mathematics.

Today, especially in higher grades and in post-secondary settings, with student learning increasingly occurring in online settings, there is a focus on web-based intelligent agents that may act as content tutors or as online discussion facilitators (Adamson, Dyke, Jang, & Rose, 2014; Tegos, Demetriadis, & Tsiatsos, 2014). AI support of online learning is especially important with the growth of Massive Open Online Courses (MOOCs), where enrollment in the most popular MOOC platforms averages over 40,000 students (Ferenstein, 2014). AI can play a role in organizing and supporting online collaboration and in assessing student learning.

Another form of educational AI, which most of us take for granted, is online search engines coupled with the tremendous amount of freely accessible online information. If we need a definition, the knowledge to complete a task, or help to understand a concept, a quick search of available online knowledge will identify a variety of text and multimedia resources to assist us.

Computational Thinking

CT in education has three instances: screen-based coding, digital tangibles (such as programmable robots and circuits), and off-screen algorithms or pseudocode. The term *computational thinking* was popularized by Wing's (2006) advocacy, "To reading, writing, and arithmetic, we should add computational thinking to every child's analytical ability" (p. 33).

Currently computational thinking in education is more as its own, isolated curriculum objective, rather than integrated with, and enriching, existing subject areas. However, there is a natural connection between computational thinking and mathematics—such as in the logical structure or in the ability to model mathematical relationships (Wing, 2008).

AI ∩ CT ∩ ME

Let us now turn to the intersection of AI, CT and ME and explore their common focus on agency, modelling and abstraction.

Agency

AI. Agency and the associated features of self-regulation and self-learning are key aspects of AI. Let's take self-driving cars as an example, where a core problem is the analysis of sensor and image data. What kind of object is in front of the car, and how should the car respond?

> It examines the images and guesses the kind of object in each image. Initially most of its guesses will be wrong. Therefore, the algorithm modifies internal parameters or parts of its structure somewhat and tries again. This process continues, discarding changes that reduce the algorithm's accuracy, keeping changes that increase the accuracy, until it correctly classifies all images. Afterward, when entirely new images are presented to the algorithm it will classify them with high accuracy. The algorithm has learned! ("Top misconceptions," 2015, para. 29)

The team of programmers designing the self-driving car could attempt to anticipate every obstacle or situation, but variations are too numerous. The car-in-action has to be able to learn from its experience and to make decisions based on that self-learning. What is also interesting is that once one car learns something from a situation, its knowledge can be immediately shared with all other cars, so that all cars learn.

CT. Student agency is a key feature of education-oriented CT environments. Building on Papert's (1980) work with Logo programming, several programming languages are available today (e.g., Scratch, available at https://scratch.mit.edu/), that offer a low floor, enabling even young children to engage with little prerequisite knowledge, and a high ceiling, providing opportunities to explore more complex relationships. As elaborated in greater detail in Gadanidis, Hughes, Minniti & White (in press) this environment offers students opportunities to abstract, automate and dynamically model concepts, to explore their relationships and to experience conceptual surprise and insight, not only by implementing pre-programmed simulations, but also by creating and editing their own, thus experiencing CT and mathematics as producers as well as consumers. For example, Figure 2 shows the Scratch code for drawing a set of circles, rotated about a point. Young students can drag and drop code blocks that snap together to model various of mathematical concepts. In such computer coding experiences, students are in control, writing personally meaningful code and exploring related problems and extensions.

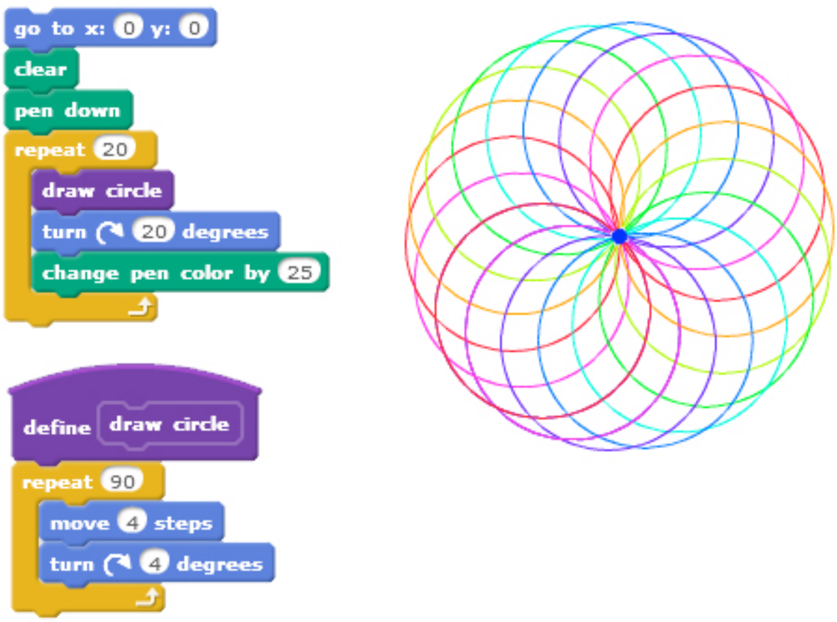

Figure 2. Creating a circles pattern in Scratch.

ME. Students' agency is also a key feature of ME theory. Burton (1999) suggests that agentic control makes a substantial difference in mathematics attitude and achievement. Schoenfeld (1987) suggests, "Many students come to believe that school mathematics consists of mastering formal procedures that are completely divorced from real life, from discovery, and from problem solving" (p. 197). Papert (1993, p. 25) adds, "I am convinced that the best learning takes place when the learner takes charge."

Modelling
AI. Developing a self-driving car involves conceptualizing models of how other cars move and react and how pedestrians interact with vehicles, to give two examples. Similarly, designing intelligent agents in education contexts, such as tutoring or online learning facilitation, requires the development of models of the subject matter and of the learners. This model-creation and the associated model-testing and model-refinement is an integral component of AI development.

CT. CT is an approach to problem solving that focuses on the logic and design of computational algorithms, or sequences of steps that can be implemented using a computer (Aho, 2012; Wing, 2006, 2008, 2011). The power of CT modelling is its dynamic nature: making a change in the computer code shows the mathematical reaction immediately. For example, changing the values of parameters in Figure 2 can cause the program to draw fewer circles or different shapes.

ME. Dynamic modelling allows students to "play" with mathematics and helps bring to life the concepts students are studying (Sinclair & Jackiw, 2009). Play naturally engages children with creative problem solving (Ginsburg, 2006) and has historically been valued in early childhood learning (Perry & Dockett, 2002; Duncan & Lockwood, 2008).

Abstraction
AI. Abstraction "plays a key role in representing knowledge and in reasoning" (Saita & Zucker, 2013, p. 2), and is an integral component of AI development. For example, in the case of the self-driving car, creating a model of "pedestrian" abstracts key attributes.

CT. Yadav et al. (2014) note that abstraction is a key element of CT. Wing (2008, p. 3717) states, "In computing, we abstract notions beyond the physical dimensions of time and space. Our abstractions are extremely general because they are symbolic, where numeric abstractions are just a special case." This process of abstraction can be seen in Figure 1, where the code used represents a variety of related cases at once.

ME. Abstraction is at the heart of mathematics. Abstraction, in the everyday sense of the word, is also a natural human activity. For example, very young children easily abstract beyond specific instances of objects and develop mental models of classes of objects, such as "cat," despite the many different sizes, colours and behaviours of cat instances.

However, as I have argued in Gadanidis (2014, 2015) the idea of engaging young students with abstraction is not widely accepted in education, primarily due to the widespread acceptance of Piaget's stages of development. Egan (2002) notes that "Piaget's ideas and overall approach absolutely dominate in education" (p. 105). Papert (1980), Egan (1997), Fernandez-Armesto (1997) and Schmittau (2005) challenge Piaget's notion that young children are not capable of abstract thinking, which Egan identifies as integral to language development. Abstraction helps students conceptualize and engage with complex problems and relationships by reducing information and detail. Wing (2011) notes that we use abstraction to better manage complexity.

A Classroom Example: Artificial Intelligence Mathematician

Agency, modelling and abstraction are integral components of AI, CT, and ME. The following is the first draft of a plan to bring all of these into play in a K-8 mathematics learning environment, all at once, by engaging students in the design and development of a numeracy intelligent agent. We have tentatively called this AI agent Artificial Intelligence Mathematician or AIM, although in the end its name will be decided by the students themselves.

Students in K-8 develop numeracy skills, ranging from a sense of number to a variety of computational procedures. The goal is not simply for students to remember definitions or algorithms, but to develop robust conceptual models and thinking skills for analyzing problem situations and deciding which methods may be most appropriate for specific situations. For example, in multiplying 26 x 257, they might use a calculator or the standard paper-and-pencil algorithm, and in multiplying 26 x 19, they might mentally multiply 26 x 20 (520) and then subtract the extra 26 (520 - 26 = 494). The solution of 26 x 19 = 26(20 - 1) also uses a form of expanded notation, the distributive property, and models that expressions such as 3(x+1) and 3x+3 are the same, thus making important numeric and algebraic connections.

Students will use unplugged CT methods, such as flowcharts or pseudocode, to design the decision making that AIM will use in responding to computation questions posed. We also plan that students will create support material to enhance the learning experience offered by AIM, by adding where they deem appropriate, text, images, videos, art and even songs they write and perform.

Student designs of AIM will initially be programmed in Scratch by one of our graduate students in computer science. Scratch allows users to also access, copy and edit the code, and we foresee that some K-8 students (especially in the higher grades) will do some of the programming. AIM will be publicly available, so family and friends as well as the wider community may engage with AIM and perhaps even offer feedback.

Engaging students with AIM, we are at once engaging them with AI, CT and ME. We are also offering them opportunities to: (a) to use their agency in the design of AIM, (b) model their mathematical thinking using CT, and (3) abstract beyond specific instances by classifying problems and their solutions.

Concluding Remarks

This paper offers a nascent exploration of the intersection of AI, CT and ME, highlighting three of their common elements: agency, modelling and abstraction. Seeing ME through the lenses of other disciplines, and recognizing that there is a significant overlap of key elements, reinforces the importance of agency, modelling and abstraction in ME and provides new contexts and tools for incorporating them in classroom practice.

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