EXTERNAL REPRESENTATIONS AND PROBLEM SOLVING COMPETENCE: DO GRAPHS IMPROVE PROBLEM SOLVING IN STUDENTS?

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Abstract
With the advent of computer technology and the popularity of the Internet as an information-gathering tool, the academic environment has changed. In today's digital age, students expect their education to include technology. Given the shift in information delivery and the expectations of students, it is important to assess how well students are able to learn when presented with different types of information. Graphs have proven to be an effective communication and presentation tool, but as technology has advanced the methods available for displaying data have multiplied. The primary goal of this study was to explore individual differences in graph comprehension. Overall results suggested that students who have high mathematical problem solving scores and are able to correctly identify the function of different displays are better able to accurately extract information from visual displays. Implications for education are discussed.

Introduction

Educators at all levels, from primary school to university, integrate technology into their teaching. When discussing literacy, computer literacy is often placed on the same level as verbal or mathematical literacy. The modern classroom is technologically rich and may include computers, interactive whiteboards, student response systems (i.e., clickers), digital media resources, and web based curricular activities. Many argue that the inclusion of technology enhances learning and leads to students who are not only better informed but who are also more effective critical thinkers and problem solvers.

As a result of technological advancements and greater exposure to interactive media, there has been a shift in students' abilities away from textual literacy towards visual literacy. Prensky (2001) argued that the rapid dissemination of technology has fundamentally changed the way students learn and, as such, the educational system has adapted to accommodate technological advancements. Educators are incorporating more visual and interactive media into learning environments (Hartman, Dziuban, & Brophy-Ellison, 2007).

Educational Value of Diagrammatic Representation

Given the increase in visual media, it is important to carefully examine how traditional and effective teaching aids (such as scientific diagrams) are used. Paivio (1986) proposed that the brain has two independent (and interconnected) systems for storing and processing information. The nonverbal system processes images (including graphical displays) and the verbal system processes language-based information. Paivio explained that, as activity spreads, associative interconnections are formed *between* the verbal and visual systems whereas referential interconnections allow activity to spread *within* the system. Through these connections related concepts lead to similar activity patterns (for example, the word *pencil* could also activate related words like *eraser* or *pen)*. Mayer and Anderson (1991) presented an updated model of this theory and applied it to student learning. This integrated, dual-code hypothesis includes connections between the verbal and visual systems; according to this model, as learning occurs, visual and verbal representations form.

According to Paivio (1986), and based on research assessing the dual-coding model, graphs are of great use in education (Mayer, 1989; Mayer & Moreno, 2002). Since the information retrieved from the display is stored and interpreted by two brain systems, ideas presented graphically should be easier to recall and, recall should include enhanced clarity. Associations between the visual and verbal components of a display result in more pathways for retrieving and analyzing information. Thus, visual representations may be an effective communication device because the information contained in a display is processed by both the verbal and nonverbal systems. This *dual coding* makes it easier to extract the information contained in a display, thus potentially increasing the ability of students to accurately understand complex information.

Trickett and Trafton (2006) suggested that graph reading involves both perceptual and spatial processing. Perceptual processing involves directly comparing graph elements and requires direct extraction of explicit information (for example, *is Bar A higher than Bar B?*). If a reader is asked to compare points on a graph separated by other elements or asked to compare different graphs, a more complex strategy, involving the spatial transformation of graphed elements, is necessary. To test the validity of these dimensions of comprehension, Hunter, Jones, Hickman, MacDonald, and Best (2007) formulated a test of graph comprehension that included read-off, spatial transformation, and prediction/interpretation questions. Overall accuracy for read-off questions was 92% suggesting that these questions required little cognitive processing or prior graph knowledge. On questions requiring spatial transformations, accuracy dropped to 71% and when participants were asked to integrate and extrapolate graphed information to predict future trends, accuracy was 48%. These findings confirm Trickett and Trafton’s suggestion that there are different levels of graph reading skills and suggest that students are able to identify variables and basic relationships but lack the skills and experience to integrate information and predict future trends.

Problem Solving and the Use of External and Internal Representations

External representations are visual representations that enhance problem solving and reasoning abilities (Cohen & Hegarty, 2007). Examples of external representations include diagrams, graphs, charts, tables, maps, and lists (Cox, 1999; Cox & Grawemeyer, 2003). Cox and Grawemeyer examined the ability of participants with differing analytical reasoning abilities to categorise, name, and use different external representations. When asked to sort 87 representations into different categories, participants produced approximately 13 categories. After categorising the representations, participants were required to name each of the sorted categories and these labels were compared to reference definitions. On this task, participants who had high analytical reasoning scores produced more accurate category labels than those who had low analytical reasoning scores. Overall, it was found that participants with high analytical reasoning scores were able to appropriately label external representations and were more competent on associated problem solving tests.

In a follow-up, Cox, Romero, du Boulay, and Lutz (2004) examined problem solving in a group of computer programmers. Participants were required to complete four external representation tasks. The first task required that the participants decide if a series of diagrams were real or *fake*. The second task was a categorisation task that tested semantic knowledge; the participants were asked to determine which of 12 categories a series of images belonged to. For the functional knowledge task, participants were instructed to select the function of the representation from 12 possible options. The final task required the participants to correctly name each representation based on 12 options. After completing the external representation tasks, participants completed two computer programming tasks to test their programming comprehension and debugging ability. Overall, results indicated a strong relationship between program comprehension and functional knowledge of external representations, suggesting that the ability to use external representations is related to problem solving ability.

Cohen and Hegarty (2007) hypothesized that the use of internal representations increases one's ability to effectively use external representations, in part because internal spatial visualisations are one component of individual spatial ability. Internal representation centres on one's ability to mentally represent spatial relationships and includes the ability to use visual spatial representations to understand and solve problems. Blajenkova, Kozhevnikov, and Motes (2006) proposed that there are two distinct types of tasks that affect performance on mental imagery tasks. Object imagery is described as "mental representations of an actual object in terms of detailed form, size, shape, colour and brightness" (p. 240). Spatial imagery involves "abstract representations of the spatial relations amongst objects, parts of objects, locations of objects in space, movements of objects and object parts and other complex spatial transformations" (p. 240). According to these researchers, mental spatial transformations are used during the problem-solving process and, as such, are of central importance to the processing of graphical displays.

Based on Trickett and Trafton's (2006) integrative theory and the relationship between internal/external visualisations and problem solving, we were interested in examining the connection between these variables and graph comprehension. If visual displays are to be effective, they must be effective for all and an understanding of underlying differences would allow educators to effectively teach graph reading skills. A long-term goal of this project is to develop strategies to enhance graph comprehension and maximize graph literacy for all students.

Method

The primary goal of this study was to explore individual differences in graph comprehension. Participants completed a test of graph comprehension (with read-off, transformation and integration questions), the object spatial imagery questionnaire (OSIQ; Kozhevnikov, Kosslyn, & Shephard, 2005), the mathematical problem solving inventory (MPI; Suwarsono, 1982, as cited in Lean & Clements, 1981), a styles of processing inventory (SOP; Childers, Houston, & Heckler, 1985) and a test designed to measure how students use external representations in problem solving (ER; see Cox et al., 2004).

Participants

Seventy participants (18 male, 52 female) were recruited; 33 participants were drawn from the introductory psychology research pool and 37 were recruited from upper-level courses. The average age of participants was 21.36 years (SD = 4.53).

Materials

The 45-item graph comprehension test was a multiple choice test based on Trickett and Trafton's (2006) three levels of comprehension. For our sample, Chronbach's alpha for the test was .709. Items for the ER test were drawn from a larger data base of items described by Cox and his colleagues (2004). In a decision task, participants were presented with 31 diagrams and asked to indicate if the diagram was *real* or *fake.* In the second task, participants were presented with 24 diagrams and had to classify and determine the function of the diagram. Chronbach's alpha for the overall test was .673. The MPI is a 14 item test designed to measure mathematical problem solving. Participants were instructed to solve the problems using diagrams to aid their thought processes. The SOP is a 22-item scale developed to assess individual preferences for using a visually oriented versus a verbally oriented form of information processing. The OSIQ is a 30-item test designed to differentiate between object and spatial imagers. Blajenkova and her colleagues (2006) reported a Chronbach's alpha of .79 for the spatial scale and .83 for the object scale.

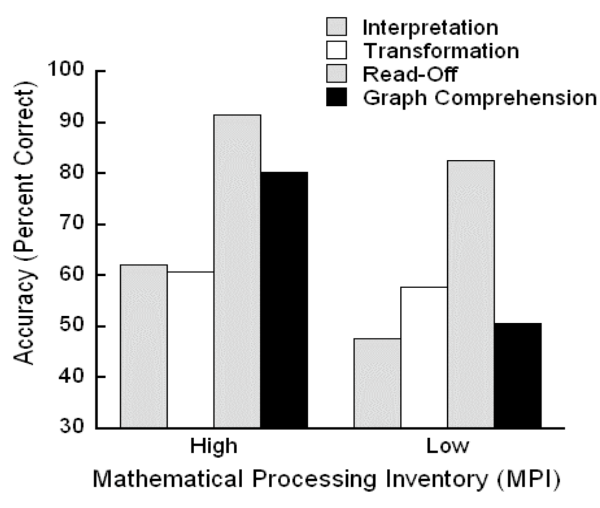
Procedure

Participants were presented with packages containing a demographics form, the Graph Comprehension Test, the MPI, the SOP, and the ERT. The OSIQ was administered by computer in the testing room. With the exception of an informed consent form and demographics questionnaire, the order of the materials was counterbalanced to reduce carryover effects.

Results

For ease of interpretation, scores on the MPI, ERT, and Graph Comprehension Test were converted into percent correct scores. Scores on the MPI were coded as *high* or *low* based on a median split. o examine overall differences in graph comprehension, a 3 (question type) x 3 (graph type) x 2 (MPI score) mixed model analysis of variance was conducted. There were statistically significant main effects for question type, $F(2, 130) = 138.44$, $p = .0001$, $\eta^2 = .68$, graph type, $F(3, 195) = 77.83$, $p = .0001$, $\eta^2 = .55$, and MPI score, $F(1, 65) = 12.89$, $p = .001$, $\eta^2 = .17$. As can be seen in Figure 1, overall graph comprehension scores were higher for those who had higher mathematical problem solving scores.

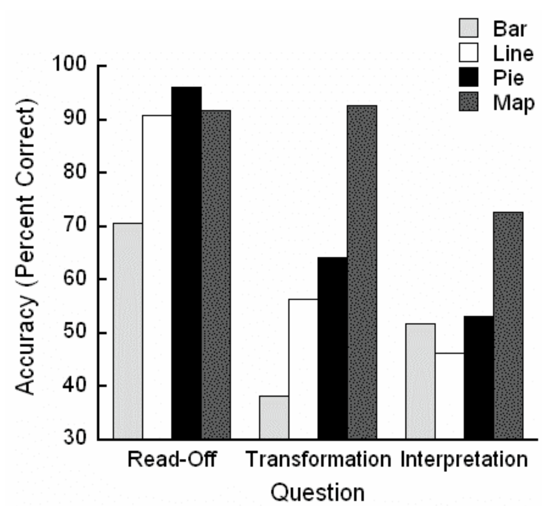
Figure 1: Graph comprehension as a function of mathematical problem solving



Using post hoc tests (in all cases, the Least Squared Difference, LSD test was used to test specific comparisons), graph comprehension was found to be significantly higher for read-off questions, intermediate for transformation questions, and lower for interpretation questions. Overall comprehension averaged 85.75% when data was presented on maps and was lower when pie charts (M = 71.21%), line graphs (M = 64.54%), and bar charts (M = 53.54%) were used. There were statistically significant differences between each of the graph types. Overall, those with high mathematical problem solving scores had higher scores on overall graph comprehension and there was a statistically significant interaction between MPI score and question type, $F(2, 65) = 4.27$, $p = .016$, $\eta^2 = .06$. Figure 1 shows that participants who were more proficient in mathematical problem solving had higher scores on interpretation and transformation questions (accuracy was similar on read-off questions).

Figure 2 shows a statistically significant interaction between graph type and question type, $F(6, 390) = 22.18$, $p = .0001$, $\eta^2 = .26$. For read-off questions, accuracy was lowest when bar graphs were presented. On transformation questions, there were statistically significant differences between each of the display types. Accuracy was highest for maps, followed by pie charts, line graphs, and bar charts. On interpretation questions, accuracy was significantly higher when maps were presented.

Figure 2: Graph comprehension as a function of graph type



The Relationship between Graph Comprehension and Internal Visualisation

Both the OSIQ and the SOP are measures of internal visualisation. As stated above, the OSIQ allows one to analyse both object and spatial imagery preferences. Based on past research (Stewart, Hunter, & Best, 2008) it was predicted that participants with high spatial ability scores would have higher graph comprehension scores. To determine the overall preference for object or spatial imagery a difference score was calculated such that positive scores indicated a preference towards object imagery and negative scores indicated a preference towards spatial imagery. Based on the OSIQ difference score, participants were classified as object imagers, spatial imagers, or as having no preference. Correlational analyses revealed no statistically significant relationships between imagery preference and graph comprehension. Further analysis using a 3 (question type) x 4 (display type) x 3 (imagery preference) mixed model ANOVA revealed no significant differences in graph comprehension as a function of imagery preference.

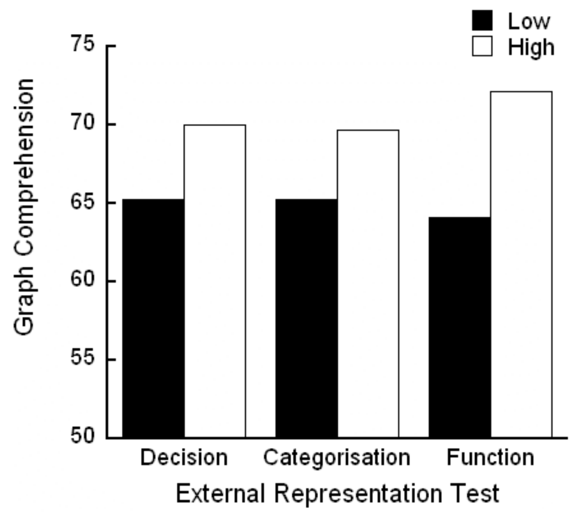
The SOP yields separate scores for visual and verbal learning preference. A difference score was calculated with positive scores indicating a preference for visual learning and negative scores indicating a preference for verbal learning. Participants were classified as having a visual preference, a verbal preference or no preference. Correlational analyses indicated that those with a visual learning preference had higher scores on transformation ($r = .257$, $p = .036$) and interpretation ($r = .256$, $p = .037$) questions when presented with bar charts. Further examination of the relationship between preferred style of processing and graph comprehension using a 3 (question type) x 4 (display type) x 3 (style of processing) mixed model ANOVA showed that style of processing did not significantly affect graph comprehension.

The Relationship between Graph Comprehension and External Visualisation

Following Cox and his colleagues (2003, 2004), three measures of external representation knowledge were used. On the decision task, accuracy was high (77.47%) regardless of MPI score. On the categorisation task overall accuracy was 69.13%; those with high MPI scores had higher (M = 73.15%) scores than those with lower (M = 64.31%) MPI scores. On the functional task, overall accuracy was lower (M = 48.30%) and those with higher MPI scores were more accurate than those with lower scores (M high = 53.58%; M low = 41.94%).

Scores on the three ERTs were grouped based on a median split and three separate one-way ANOVAs were used to examine the effect that external representation knowledge had on overall graph comprehension. As can be seen on Figure 3, graph comprehension was similar for all participants on the decision and categorisation tasks. However, those who had higher scores on the function task had higher overall graph comprehension, $F(1, 63) = 10.46$, $p = .002$, $\eta^2 = .14$.

Figure 3: Graph comprehension scores as a function of external representation



Prediction of Graph Comprehension Scores

To determine the effects of the MPI, ER (decision, categorise, function), OSIQ and SOP variables on overall graph comprehension a stepwise linear regression analysis was conducted. Analysis revealed that the MPI and the ER (function) together significantly predicted overall graph comprehension ($F(2,61) = 21.06$, $p = .0001$). The total model accounted for 40.8% of the variance, with the MPI accounting for 29.3% of the variance and the ER (function) contributing an additional 11.5%. All other variables were not statistically significant predictors.

Discussion

One purpose of the current study was to examine graph comprehension in university students. Overall, graph comprehension was highest when participants were asked to simply extract a single variable from a display. When graph reading involved making spatial transformations or broader interpretations about the graphed data, comprehension was lower. These findings support previous research (see Stewart et al., 2008; Trickett & Trafton, 2006) suggesting that there are different levels of graph comprehension and also lend support to the conceptualisation of graph comprehension as involving both perceptual and cognitive processes. When presented with read-off questions, participants used basic perceptual processes to decode the graphed variables and extract the relevant piece of information. When answering transformation and interpretation questions, participants were required to draw on their individual graph schema and had to rely on prior knowledge to answer specific questions.

Overall comprehension was higher when data was presented on maps and pie charts and was lower when line graphs and bar charts were used. The accuracy differences are likely due to the types of information typically displayed on these graph types and the complexity of different formats. For example, if asked to combine different variables on a bar graph, a reader would have to mentally move and combine different bars. When presented with the same question based on pie chart information, different pieces would have to be combined. However, given the fact that most readers know that each piece is a proportion of the whole, the combination is likely to be easier. It is possible that bar charts and line graphs typically present more complex information and thus, the ability of participants to extract information from these types of displays is lower.

Implications for Education

Although many studies have examined graph comprehension, most (see Cleveland & McGill, 1985; Pinker, 1990) have focused on determining the underlying perceptual and cognitive processes. One of our goals was to examine the correlates of graph comprehension; specifically, we were interested in determining how individual problem solving ability, mental imagery preferences, learning style preferences, and external representation knowledge affected comprehension. Interestingly, the best predictors of graph comprehension were mathematical problem solving and the ability to identify the function underlying different display types.

Cox and his colleagues (2003, 2004) found that computer programming skill was related to the ability to correctly identify the function of different displays. We found statistically significant correlations between graph comprehension and each of the external representation tasks. When participants were asked to determine the function of a display, the correlation with graph comprehension was higher (r = .53) than when they were asked to decide about the validity of a display (r = .34) or categorise a display (r = .28). Pinker (1990) suggested that each graph reader has an individual graph schema that is activated when they extract information from different graphs. The graph schema is very personal and depends upon prior knowledge. The current results lend proof to the conceptualisation of a graph schema. Participants with a more detailed graph schema would have more experience reading graphs and would be more likely to be familiar with a wide variety of displays and the function of different displays.

In the current study, graph comprehension did not differ according to imagery preferences or learning style. Using a similar graph comprehension test, Stewart and her colleagues (2008) reported that spatial imagers were better able to extract information from different displays. In that study, spatial and object imagery were assessed using tests developed by Kozhevnikov, and her colleagues (2005) to measure object and spatial imagery. The object imagery test was designed to assess the ability to recognize and identify objects presented in a degraded or fuzzy picture. The spatial imagery test included box folding tasks, cube rotation and tasks requiring the transformation of block orientation. In the current study, a self- report inventory was used to assess object and spatial imagery and no differences in comprehension due to imagery preference were found. It is likely that tests that require participants to perform different tasks are more sensitive to differences than a self-assessment inventory but future studies should further examine the relationship between these variables.

Overall, the current results are positive. Mathematical problem solving and external representation knowledge are skills that can be taught. The current results suggest that all students, regardless of their spatial ability, learning styles, or imagery preferences can effectively extract information from visual representations. These results also suggest that a focus on graphical literacy in the classroom could lead to increased ability to understand information. In this digital age students expect information to be presented in a variety of formats. Instructors regularly present visual information and integrate web-based material into their courses. If this information is to be effective, it must be carefully designed and specific learner characteristics must be taken into account.

General Conclusions

As advocates for both high-level teaching and learning environments it is essential that we understand the power of diagrams. It is important that we understand that diagrams in curricula should not only be used to convey information but also as a tool that students can use to help them think critically outside of the classroom. Enhancing graph literacy will allow students to extract relevant information from a graph and make informed interpretations about the graphed data. Additionally, students able to design graphics become better learners, problem solvers, and critical thinkers. Future research on diagrams should focus on developing teaching aids and instruction guides that help students improve their graph literacy.

Given the fact that we are becoming a “visual” culture and many current teaching strategies focus on web-based learning and visual presentation (i.e., PowerPoint presentations, animated tutorials), it is important to carefully examine whether these types of learning environments facilitate the learning process. Experimental evidence focusing on the empirical validation of visual learning tools would allow us to design educational programs that best meet the needs of learners (children and adults) and allow us to develop quality educational programs for all learners. These results have important implications to e-learning programs and could lead to the development of specific programs geared towards specific groups of learners. In addition, many private sector businesses develop and implement training programs for their employees and research such as this would allow businesses to develop optimal training programs.

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